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Form Approved OBM No. 0704-0188

Public re[illegible] ... per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaini[illegible] ... Send comments regarding this burden or any other aspect of this collection of information, including suggestions for reduc[illegible] ... tion Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office ... ashington, DC 20503.

1. Agen[illegible]	1993	3. Report Type and Dates Covered. Final - Proceedings

4. Title and Subtitle.
An Iterative T-Matrix

5. Funding Numbers.
Contract
Program Element No. 0601153N
Project No. 03202
Task No. 350
Accession No. DN255011
Work Unit No. 12211B

6. Author(s).
M. F. Werby and Guy V. Norton

7. Performing Organization Name(s) and Address(es).
Naval Research Laboratory
Center for Environmental Acoustics
Stennis Space Center, MS 39529-5004

8. Performing Organization Report Number.
PR 91:098:221

9. Sponsoring/Monitoring Agency Name(s) and Address(es).
Naval Research Laboratory
Basic Research Management Office
Stennis Space Center, MS 39529-5004

10. Sponsoring/Monitoring Agency Report Number.
PR 91:098:221

11. Supplementary Notes.
Published in IMACS, Computational Acoustics.

12a. Distribution/Availability Statement.
Approved for public release; distribution is unlimited.

12b. Distribution Code.

13. Abstract *(Maximum 200 words).*

The T-matrix method, a technique invented by Peter Waterman, (1965, 1969), 1977, 1980) has proven to be valuable for calculating classical scattering from bounded targets. The method is useful for acoustical, electromagnetic and elastic scattering and is based on coupled boundary integral equations and partial wave expansions of relevant physical quantities. Upon truncation of the partial wave expansion, which are in principle infinite, a system of equations is derived that can be expressed in terms of finite matrices. This system in turn leads to an expression that maps the incident field onto the scattered field. The outcome is a powerful algorithm that yields essentially exact results. The point at which the partial wave series is truncated is always an issue in using the method. If too few terms are included, the results will be very inaccurate. Conversely, if many terms beyond that required for convergence are included, at best the calculation will be needlessly time consuming and, at worst, can lead to numerical instability due to numerical round-off errors. What would be ideal is a technique that leads to optimum truncation, that is, the point just beyond which convergence is achieved. This type of technique can be obtained by a method based on starting with a "small" t-matrix and in which we build up the dimensions term by term via simple vector operations until convergence is achieved. The algorithm is described and results are illustrated for some examples

9[illegible] 030

14. Subject Terms.
Acoustic scattering, shallow water, waveguide propagation

15. Number of Pages. 8

16. Price Code.

17. Security Classification of Report.	18. Security Classification of This Page.	19. Security Classification of Abstract.	20. Limitation of Abstract.
Unclassified	Unclassified	Unclassified	SAR

ISSN 1067-0688
Volume 2, 1993
Section B

Mathematical Modelling and Scientific Computing

Affiliated with the International Association for Mathematical and Computer Modelling

EDITOR-IN-CHIEF
Xavier J. R. Avula

Mathematical Modelling and Scientific Computing in Science and Technology

THE EIGHTH INTERNATIONAL CONFERENCE
College Park, Maryland, U. S. A. April 1991

PRINCIPIA SCIENTIA
St. Louis · Madison

Math Modelling and Sci. Computing, Vol. 2, pp 792-797, 1993
Principia Scientia. Printed in U.S.A.

AN ITERATIVE T-MATRIX

M. F. WERBY and GUY V. NORTON

Naval Oceanographic and Atmospheric Research Laboratory
Theoretical Acoustics, Stennis Space Center, MS 39529-5004

ABSTRACT

The T-matrix method, a technique invented by Peter Waterman, (1965, 1969, 1971, 1977, 1980) has proven to be valuable for calculating classical scattering from bounded targets. The method is useful for acoustical, electromagnetic and elastic scattering and is based on coupled boundary integral equations and partial wave expansions of relevant physical quantities. Upon truncation of the partial wave expansion, which are in principle infinite, a system of equations is derived that can be expressed in terms of finite matrices. This system in turn leads to an expression that maps the incident field onto the scattered field. The outcome is a powerful algorithm that yields essentially exact results. The point at which the partial wave series is truncated is always an issue in using the method. If too few terms are included, the results will be very inaccurate. Conversely, if many terms beyond that required for convergence are included, at best the calculation will be needlessly time consuming and, at worst, can lead to numerical instability due to numerical round-off errors. What would be ideal is a technique that leads to optimum truncation, that is, the point just beyond which convergence is achieved. This type of technique can be obtained by a method based on starting with a "small" T-matrix and in which we build up the dimensions term by term via simple vector operations until convergence is achieved. The algorithm is described and results are illustrated for some examples.

KEYWORDS

T-matrix; partial wave functions; convergence; acoustic; scattering.

INTRODUCTION

The appropriate number of basis functions must be used when generating a T-matrix; if too few are used, the T-matrix will not converge (Waterman, 1965, 1969, 1971, 1977, 1980; Wall, 1980; Werby and Chin-Bing, 1985; Werby, 1991). When generating a T-matrix for well-known shapes, a few rules of thumb usually pertain to the number of basis functions to use, but not for arbitrary shapes. In the past if the T-matrix was insufficiently exact, the T-matrix would have to be regenerated with more terms. This regeneration, however, did not guarantee that the new T-matrix would converge. With this in mind, an iterative algorithm is created so that there will be no guessing as to how many basis functions is needed to ensure convergence. This paper is divided into the following sections. We will present the theory behind the procedure (starting with an nth order T-matrix), describe what occurs at convergence, and show some numerical results.

THEORY

The iterative procedure is based on the method of bordering (Faddeeva, 1959). In general, we start with a (n x n) matrix **T**. Another matrix of order (n+1 x n+1) can be constructed based on the original matrix **T**. It is thus possible to build any order matrix by this procedure, since the expanded matrix **T** order (n+1 x n+1) can be used as the next starting point to obtain the matrix **T** of order (n+2 x n+2). Let us then begin with an nth order T-matrix expressed as,

$$T_{nn} = -(\mathrm{Re}Q_{nn})Q_{nn}^{-1}, \tag{1}$$

where Re means the regular part of **Q**.

The matrix Q in general has the following form,

$$Q_{nn} = k\int_\sigma \left[\varphi_n \frac{\partial\, Re\,\varphi_n}{\partial n} - Re\,\varphi_n \frac{\partial \varphi_n}{\partial n}\right] ds \qquad (2)$$

where φ is the basis function and k is the wavenumber. We can specify either Neumann or Dirichlet boundary conditions on the object and Eq. (2) will become either

$$Q_{nn} = -k\int_\sigma Re\,\varphi_n \frac{\partial \varphi_n}{\partial n}\, ds \qquad (3)$$

$$ReQ_{nn} = -k\int_\sigma Re\,\varphi_n \frac{\partial\, Re\,\varphi_n}{\partial n}\, ds$$

with a Neumann boundary condition or

$$Q_{nn} = k\int_\sigma \varphi_n \frac{\partial\, Re\,\varphi_n}{\partial n}\, ds \qquad (4)$$

$$ReQ_{nn} = k\int_\sigma Re\,\varphi_n \frac{\partial\, Re\,\varphi_n}{\partial n}\, ds$$

with a Dirichlet boundary condition. In general, if we expand the T-matrix of order (n x n) to an order of (n+1 x n+1), Eq. (1) becomes

$$T_{n+1,n+1} = -(ReQ_{n+1,n+1})Q^{-1}_{n+1,n+1}, \qquad (5)$$

where

$$Q_{n+1,n+1} = \begin{pmatrix} Q_n & \vec{c}_{n+1} \\ \vec{r}_{n+1} & a_{n+1,n+1} \end{pmatrix} \qquad (6)$$

and

$$\vec{r}_{n+1} = \left(a_{n+1,1},\cdots,a_{n+1,n}\right), \qquad \vec{c}_{n+1} = \begin{pmatrix} a_{1,n+1} \\ \vdots \\ a_{n,n+1} \end{pmatrix} \qquad (7)$$

where $\vec{r}$ is a row vector and $\vec{c}$ is a column vector and $a_{n+1,n+1}$ is a scalar. Now Q^{-1} is expressed as

$$Q^{-1}_{n+1,n+1} = \begin{pmatrix} P_{n,n} & \vec{\xi}_{n+1} \\ \vec{q}_{n+1} & \left(b_{n+1,n+1}\right)^{-1} \end{pmatrix}, \qquad (8)$$

where P is an (n x n) matrix, $\vec{q}$ and $\vec{\xi}$ are vectors and $b_{n+1,n+1}$ is a scalar. They have the following expression;

$$P_{n,n} = Q^{-1}_{n,n} - Q^{-1}_{n,n}\,\vec{c}_{n+1}\vec{q}_{n+1}$$

$$\vec{q}_{n+1} = -\frac{\vec{r}_{n+1}\,Q^{-1}_{n,n}}{b_{n+1,n+1}}$$

$$\vec{\xi}_{n+1} = -\frac{Q^{-1}_{n,n}\,\vec{c}_{n+1}}{b_{n+1,n+1}}$$

$$b_{n+1,n+1} = a_{n+1,n+1} - \vec{r}_{n+1}\,Q^{-1}_{n,n}\,\vec{c}_{n+1}.$$

Finally we can rewrite Eq. (5) as

$$T_{n+1,n+1} = -\mathrm{Re}\begin{pmatrix} Q_{n,n} & \tilde{c}_{n+1} \\ \tilde{t}_{n+1} & a_{n+1,n+1} \end{pmatrix}\begin{pmatrix} P_{n,n} & \tilde{s}_{n+1} \\ \tilde{q}_{n+1} & b_{n+1,n+1} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{pmatrix}, \tag{9}$$

where

$$\beta_1 = T_{n,n}\left(1 - \frac{\tilde{c}_{n+1}\tilde{t}_{n+1}Q_{n,n}^{-1}}{b_{n+1,n+1}}\right) + \mathrm{Re}\,\tilde{c}_{n+1}\tilde{t}_{n+1}Q_{n,n}^{-1}$$

$$\beta_2 = \frac{\mathrm{Re}Q_{n,n}Q_{n,n}^{-1}\tilde{c}_{n+1}}{b_{n+1,n+1}} - \frac{\mathrm{Re}\,\tilde{c}_{n+1}}{b_{n+1,n+1}}$$

$$\beta_3 = -\mathrm{Re}\,\tilde{t}_{n+1}Q_{n,n}^{-1}\left(1 - \frac{\tilde{c}_{n+1}\tilde{t}_{n+1}Q_{n,n}^{-1}}{b_{n+1,n+1}}\right) + \frac{\mathrm{Re}\,\tilde{t}_{n+1}Q_{n,n}^{-1}a_{n+1,n+1}}{b_{n+1,n+1}}$$

$$\beta_4 = \frac{\mathrm{Re}\,\tilde{t}_{n+1}Q_{n,n}^{-1}\tilde{c}_{n+1}}{b_{n+1,n+1}} - \frac{\mathrm{Re}\,a_{n+1,n+1}}{b_{n+1,n+1}}.$$

Using this algorithm has several advantages. As already mentioned the algorithm takes the guesswork out of choosing the number of basis functions. The T-matrix is conditioned as it is built; this technique relaxes some of the complications associated with ill-conditioning, which is sometimes encountered for various objects. Finally, inspection of the theory shows that the addition of additional columns and rows is an operation involving either a matrix and a vector or two vectors, which if performed on a vector machine would be fast and economical.

DISCUSSION OF CONVERGENCE

Partial wave series solutions of scattering and rearrangement collisions have long been used in quantum scattering. There are two main reasons for doing so. First, it is much easier to solve for each of the partial wave solutions, which reduce from a three-dimensional problem to that of a series of one-dimensional problems for central forces, or in general for problems in the which angular contribution can be separated from the radial part. Second, convergence is generally expected at some value of angular momentum due to the centrifugal repulsion effect. What this means is, that for some value of ka where k is the wave number and a is some characteristic dimension of the object (the radius for spheres, for example), at some point the centrifugal contribution, which occurs in a partial wave solution (namely $l(l+1)/r^2$), is greater than the component $(ka)^2$. Thus the contribution from that centrifugal term is repulsive and causes little overlap with the target when it dominates.

Figure 1 illustrates this point for several incident partial waves. As evident from the figure, when the partial wave number increases, there is a resulting decrease in the overlap with the target. For suitably low values of l, the ka component dominates, so the interaction region is significant. The question for what value of l the centrifugal component will dominate has been answered for spheres by using the WKB approximation. It is generally agreed that a value of l=2 ka is large enough for the series to cut off. For spheroids the situation is more complicated, since the system of partial wave solutions are coupled and since the overlap of the incident partial waves with the surface displacement terms is rather complicated.

For the spherical case there is mode coupling of the surface boundary terms with each partial wave, and this corrupts the simple picture we have for spheres. Figure 2 illustrates the case of convergence for several spheroids with varying aspect ratios that deviate between 2 and 18 for a kL/2 fixed at 6, where L is the object length and k the total wavenumber ($k=2\pi/\lambda$). We use a low accuracy T-matrix code so that numerical difficulties will be accentuated. For low aspect ratio targets, convergence is maintained with increasing partial wave terms. As the number of partial wave terms increase, more terms are required for convergence because of an overlap effect with the surface expansion terms (Werby and Chin-Bing, 1985; Werby, 1991). Further, the point at which the answer is no longer accurate is due to ill-conditioning that results from high aspect ratio targets which occurs earlier; thus, the "window of convergence" narrows and makes the problem of convergence more urgent with increasing aspect ratio. The method outlined in the previous section not only obviates the problem of convergence but also appears to condition the matrix as well. Alternative methods exist that allow problems to be solved based on the coupled equations of

Waterman for elastic targets (Werby and Green, 1983) and from the point of view of an eigenvalue problem (Werby *et al.* 1986). The problem of a target in a wave guide has been discussed extensively in a recent article by Norton and Werby (1991).

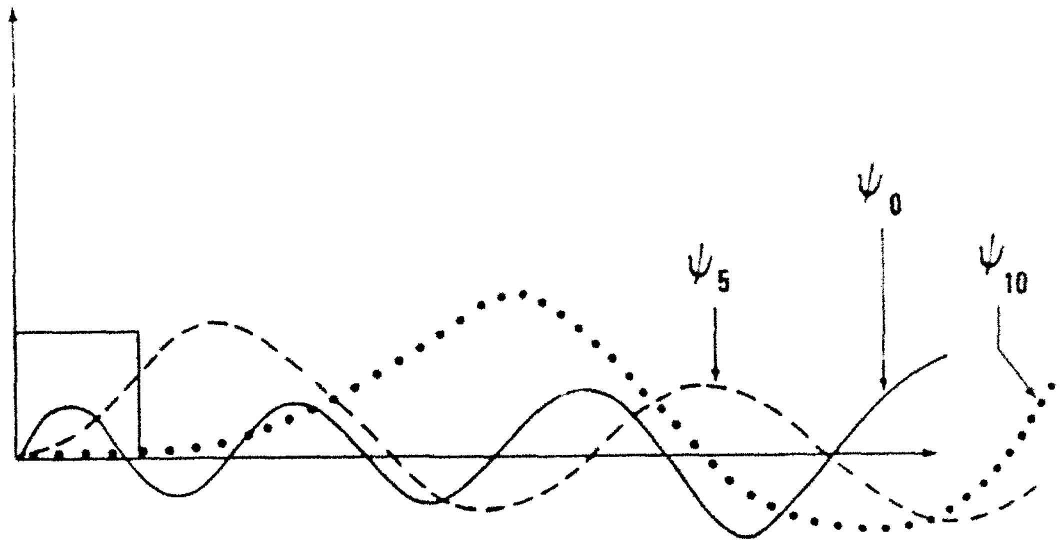

Fig. 1. Comparison of the overlap with the target for several incident partial waves.

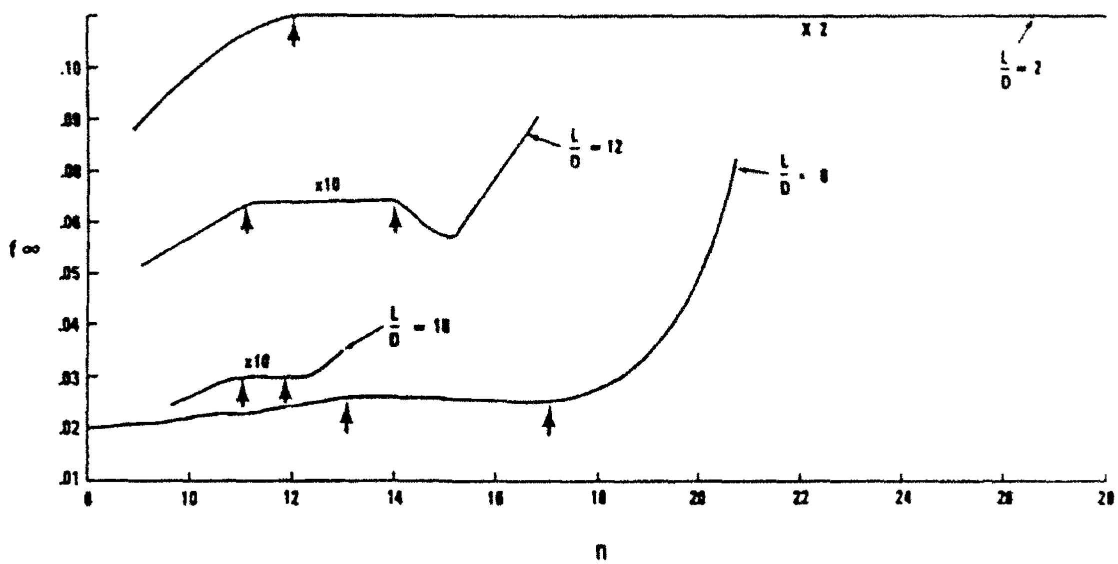

Fig. 2. Form function vs. matrix order for various aspect ratio targets. L is the length and D is the diameter of the target. The region between arrows represent the "window of convergence."

APPLICATION TO RIGID TARGETS

There are two classes of targets for impenetrable problems, i.e., soft and hard scatterers. They do not support body resonances; therefore, we examine acoustic quantities appropriate for such non-resonant targets as circumferentially diffracted, or creeping, waves. These waves arise when scattering end-on from a spheroid in which one the return signal is observed at the origin of the signal. There are two competing mechanisms. One arises from specular scattering (geometric) and the other arises from the creeping waves. The result is a coherent effect in which the two

waves add at some point constructively leading to a maximum value when they are in phase and destructively leading to a minimum when they are out of phase. The values of the incident wavefield frequency are expressed using the dimensionless quantity kL/2. Figure 3 shows a spheroid of aspect ratio (a) 4:1 (b) 8:1 and (c) 16:1. The more pronounced dips with increasing aspect ratio is due to the greater grazing angular region which gives rise to the interference phenomena for higher aspect ratio targets.

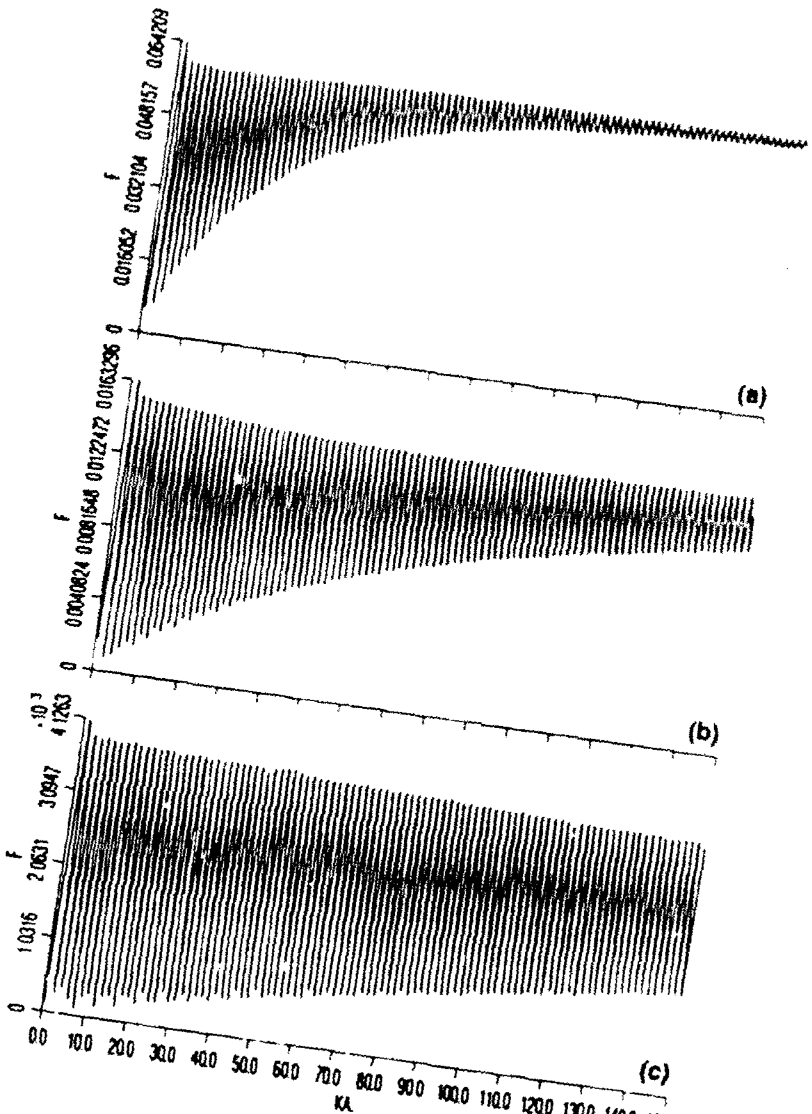

Fig. 3. Backscatter from a spheroid of aspect ratio of a) 4 to 1; b) 8 to 1; and c) 16 to 1 for kL/2 = 0 to 150.

Bistatic angular distributions correspond to measurement of a scattered field at any point in space for some incident wave fixed relative to some source-object orientation. In Fig. 4 we examine a rigid spheroid of aspect (length-to-width) ratio of 30:1. Figure 4a represents scattering from the object along the axis of symmetry (end-on) and Fig. 4b represents 90 degrees relative to the symmetry axis (broadside). The value of kL/2 in Fig. 4 is 200, which implies that the object is about 70 wavelengths long and thus is in the intermediate-to-high frequency region, where neither low nor high frequency approximations apply. In all figures, frequency is sufficiently high that wave diffraction effects are significant in the forward scattering direction.

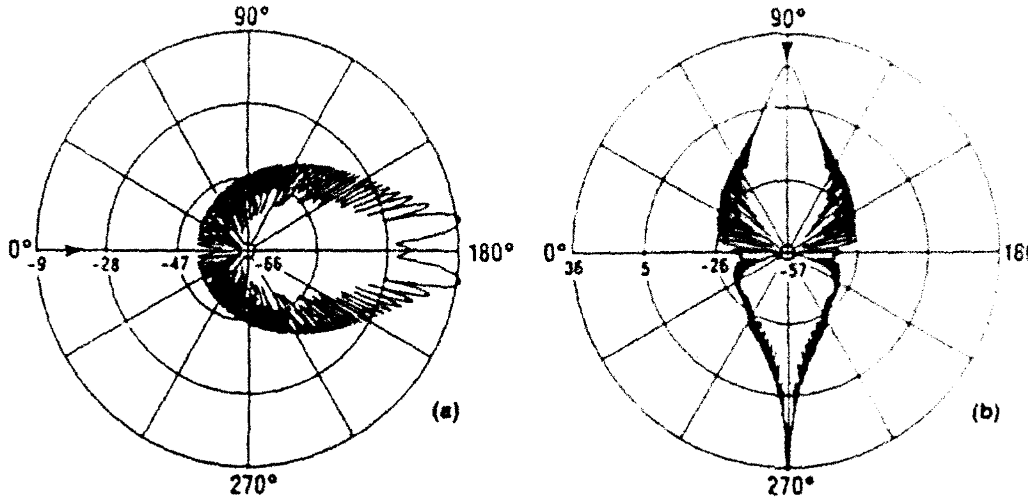

Fig. 4. a) Bistatic scattering from 30 to 1 aspect ratio rigid spheroid end on; b) broadside incidence for kL/2=200.

SUMMARY

We have described an iterative method that produces an (n+m x n+m) matrix from an (n x n) matrix. With the use of an illustration we described what must occur in order to produce convergence. Finally, we presented examples of scattering results achieved using this technique.

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